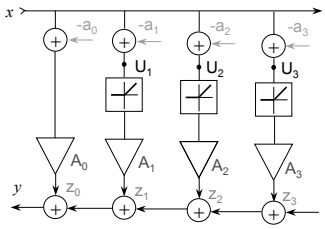
SIGNALI I SUSTAVI

AUDITORNE VJEŽBE 2

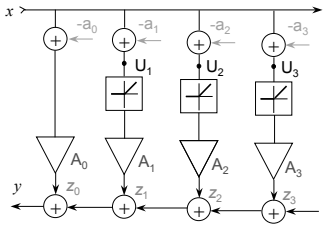
LS&S
FER-ZESOI

Aproksimacija U/I karakteristike linearnim segmentima



- Neka je:
  - $a_0 < a_1 < a_2 < a_3$,
- Onda vrijedi:
  - $U_1 = x - a_1$,
  - $U_2 = x - a_2 < U_1$,
  - $U_3 = x - a_3 < U_2$,

Aproksimacija U/I karakteristike linearnim segmentima



Analiziramo segmente:

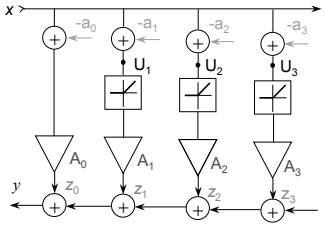
1) $x < a_1$, $x - a_1 < 0$,

$U_1 < 0$, $U_2 < 0$, $U_3 < 0$,

$\Rightarrow z_1 = z_2 = z_3 = 0$,

$y = z_0 = A_0 (x - a_0)$.

Aproksimacija U/I karakteristike linearnim segmentima



2) $a_1 < x < a_2$

$U_1 > 0,\ U_2 < 0 \Rightarrow U_3 < 0,$

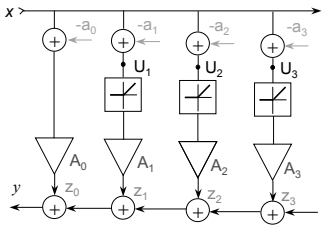
$\Rightarrow z_2 = z_3 = 0,$

$y = z_0 + z_1 =$

$= A_0\,(x - a_0) + A_1\,(x - a_1),$

$= (A_0 + A_1)x - (A_0a_0 + A_1a_1).$

Aproksimacija U/I karakteristike linearnim segmentima



3) $a_2 < x < a_3$

$U_1 > 0,\ U_2 > 0,\ U_3 < 0,$

$\Rightarrow z_3 = 0,$

$y = z_0 + z_1 + z_2 =$

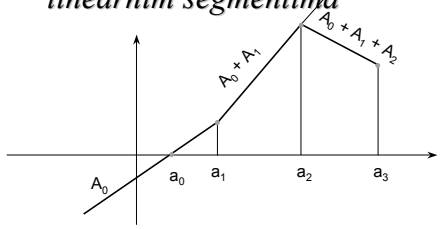
$= A_0\,(x - a_0) + A_1\,(x - a_1)$

$+ \ A_2\,(x - a_2) =$

$= (A_0 + A_1 + A_2)\,x -$

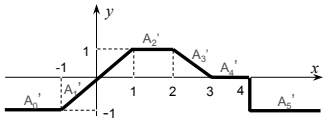
$(A_0a_0 + A_1a_1 + A_2a_2).$

Aproksimacija U/I karakteristike linearnim segmentima



- a_1, a_2, a_3 - točke loma (promjena nagiba).
- a_0 nije točka loma.

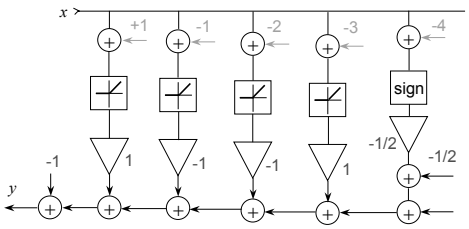
Primjer 1. Realizirati sustav s ulazno-izlaznom karakteristikom prema slici



Točke loma:

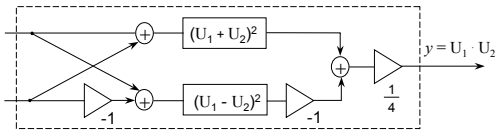
- $x < a_1 = -1 \quad y = -1,$
- $a_1 = -1 \quad A'_0 = A_0 \quad A_0 = 0,$
- $a_2 = 1 \quad A'_1 = 1 = A_0 + A_1 \quad A_1 = 1,$
- $a_3 = 2 \quad A'_2 = 0 = A'_1 + A_2 \quad A_2 = -1,$
- $a_4 = 3 \quad A'_3 = -1 = A'_2 + A_3 \quad A_3 = -1,$
- $a_5 = 4 \quad A'_4 = 0 = A'_3 + A_4 \quad A_4 = 1,$
- $A'_5 = A'_4 \quad A_5 = 0.$

Primjer 1. - nastavak



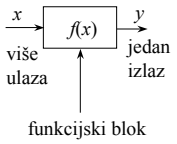
Zadatak 1.

- Treba realizirati blok za množenje.
- Blok za kvadriranje realizira se pomoću bloka za aproksimaciju U/I.
- $U_1 \cdot U_2 = 1/4\,[(U_1 + U_2)^2 - (U_1 - U_2)^2].$



Bezmemorijski kontinuirani sustavi

- To su sustavi koji su realizirani od elemenata kao što su:
  - Pojačalo,
  - Zbrajalo,
  - Množilo,
  - Prag.



Pravila spajanja:

1) Nije dozvoljeno spajanje izlaza funkcijskih blokova.
2) Svaki ulaz u funkcijski blok mora biti spojen na izlaz od nekog od ostalih funkcijskih blokova ili predstavlja ulaz u cijeli sustav.
3) Izlaz samo jednog funkcijskog bloka je izlaz iz sustava.

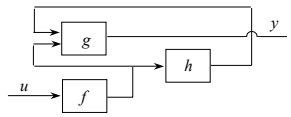
Bezmemorijski kontinuirani sustavi

- Podjela bezmemorijskih kontinuiranih sustava:
  - Eksplicitni sustavi su sustavi za koje se može napraviti sortirana lista.
  - Implicitni sustavi su sustavi za koje se ne može napraviti sortirana lista (sustavi s povratnom vezom).

Primjer 1.

- Za svaki od sustava napisati spojnu listu i provjeriti da li se lista može sortirati.

a)



$g : f, h$
$h : f$
$f : u$

spojna lista

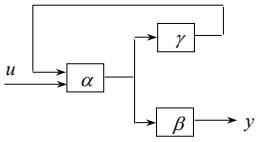
$f : u$
$h : f$
$g : f, h$

sortirana spojna lista

EKSPLICITNI SUSTAV

Primjer 1. - nastavak

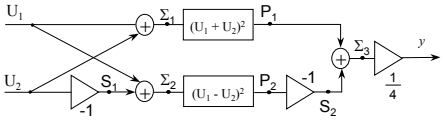
b)



- $\alpha : u, \gamma,$
- $\beta : \alpha,$
- $\gamma : \alpha.$

Ne možemo sortirati listu
Sustav je IMPLICITAN.

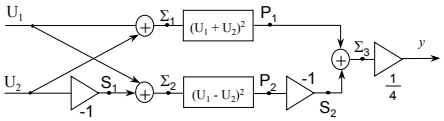
Primjer 2: Za multiplikator napisati spojnu listu i provjeriti da li se lista može sortirati.



Nesortirana lista:

- $\Sigma_1 : U_1, U_2,$
- $P_1 : \Sigma_1,$
- $\Sigma_3 : P_1, S_2.$
- $y : \Sigma_3,$
- $S_1 : U_2,$
- $\Sigma_2 : U_1, S_1,$
- $P_2 : \Sigma_2,$
- $S_2 : P_2.$

Primjer 2. - nastavak

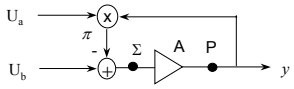


Sortirana lista:

- $\Sigma_1 : U_1, U_2$,
- $P_1 : \Sigma_1$,
- $S_1 : U_2$.
- $\Sigma_2 : U_1, S_1$,
- $P_2 : \Sigma_2$,
- $S_2 : P_2$,
- $\Sigma_3 : P_1 , S_2$,
- $y : \Sigma_3$.

Primjer 3.

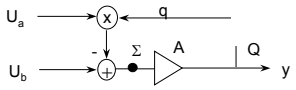
- Treba realizirati sustav koji će kao izlaz dati kvocijent dvaju ulaza.
- $y = U_b / U_a$



- $\pi : U_a, P$,
- $\Sigma : \pi, U_b$,
- $P : \Sigma$,
- Lista se ne može sortirati

$\Rightarrow$ Implicitni sustav.

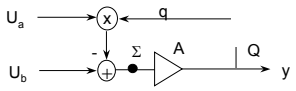
Primjer 3. -nastavak



- Uvedemo q = Q umjesto P, q dodatni ulaz
- $\pi : U_a , q$,
- $\Sigma : U_b , \pi$,
- $Q : \Sigma$,
- $y : Q$.

sortirana lista

Primjer 3. -nastavak



- Napišimo funkcijske ovisnosti:
- $\pi = U_a \cdot q$,
- $\Sigma = -\pi + U_b$,
- $Q = A \cdot \Sigma$,
- $y = Q$,
- a znamo da je $q = Q$.

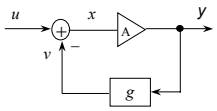
Primjer 3. - nastavak

- $q = Q = A \cdot \Sigma = A(-\pi + U_b)$,
- $q = A(-U_a \cdot q + U_b) = AU_b - AU_a \cdot q$,
- q se javlja s obje strane jednadžbe, tj.
- $q = f(U_b, U_a, q)$.
- Izrazimo izlaz y:
- $y = Q = q = A(-U_a \cdot q + U_b)$, (1)
- $y = \phi(U_b, U_a, q)$.

Primjer 3. - nastavak

- iz (1) slijedi $y = -AU_a\, y + AU_b$, (2)
- $y / (AU_a) = -y + U_b / U_a$ (nakon dijeljenja s AU_a),
- $y = U_b / U_a - y / (AU_a)$.
- Kada pojačanje $A \to \infty \Rightarrow y \to U_b / U_a$.
- Što kada je $U_a = 0$? Iz (2) vidimo da slijedi:
- $y = AU_b$, a ako $A \to \infty$ onda i $y \to \infty$.
- U praksi se y "zalijepi" na napon napajanja.
- Rezultat: od množila dobili smo djelilo (realizacija inverzne operacije) !!!

Primjer 4.: Treba realizirati sustav koji će omogućiti realizaciju inverzne funkcije



- x : u, v,
- y : x,
- v : g.

Ne može se sortirati.
IMPLICITNI SUSTAV

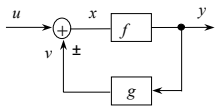
Primjer 4. - nastavak

- $x = u - v$,
- $y = \mathrm{A}x$,
- $v = g(y)$,
- $y = \mathrm{A}\,(u - v) = \mathrm{A}u - \mathrm{A}\,g(y)$.
- Izlaz ne možemo eksplicitno izraziti, ali
- možemo ulaz eksplicitno izraziti kao funkciju izlaza:
- $u = y / \mathrm{A} + g(y)$.

Primjer 4. - nastavak

- $u = y / \mathrm{A} + g(y)$.
- Ako je A>> ,
- $u \cong g(y)$ ili $y \cong g^{-1}(u)$,
- Navedeni sklop realizira inverznu funkciju od g !!!

Primjer 5. - Sustav s povratnom vezom



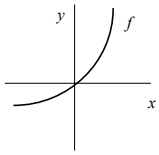
- $y = f(x)$,
- $v = g(y)$,
- $x = u \pm v$,
- $+$ pozitivna povratna veza,
- $-$ negativna povratna veza.

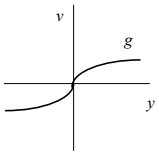
Primjer 5. - nastavak

- Izrazimo ulaz:
- $u = x \mp v \qquad = f^{-1}(y) \mp g(y)$.
- Što je inverzna funkcija sustava s povratnom vezom tj.
- $y = F(u); \qquad F = (f^{-1} \mp g)^{-1}$.
- Naravno, pretpostavka je da f i F imaju inverzne funkcije.
- Grafički se F može pronaći i kada to nije slučaj, odnosno kada su f^{-1} i F^{-1} relacije.

Primjer 5. - nastavak

- Neka je

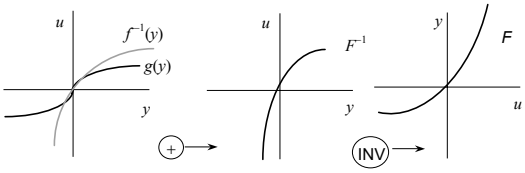




Primjer 5. - nastavak

- Sustav s negativnom povratnom vezom imat će karakteristiku koja se dobije iz:

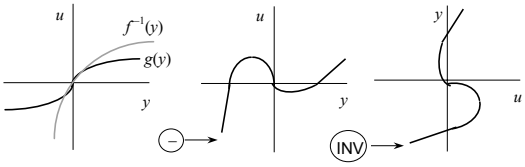
$$u = f^{-1}(y) + g(y).$$



Primjer 5. - nastavak

- Sustav s pozitivnom povratnom vezom je:

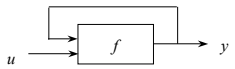
$$u = f^{-1}(y) - g(y).$$

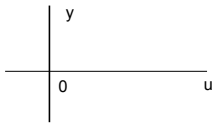


Primjer 5. - nastavak

- U drugom slučaju rezultat nije funkcijski, već relacijski blok.

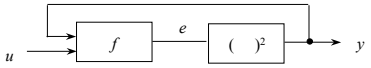
Implicitni sustavi zadani relacijama - Primjer 1.





- $f(u, y) = u + y$,
- $y = f(u, y) = y + u$,
- za $u = 0$ y je proizvoljan,
- za $u \neq 0$ nemogući izlaz,
- ne možemo realizirati sa $y = \phi(u)$.

Primjer 2.



- $f(u, y) = u + y$,
- $y = e^2 = (u + y)^2$,
- $y = u^2 + 2uy + y^2$,
- $y^2 + (2u - 1)y + u^2 = 0$.

$$y = \left(-u + \frac{1}{2}\right) \pm \sqrt{\left(-u + \frac{1}{2}\right)^2 - u^2},$$

za $\left(u - \frac{1}{2}\right)^2 \geq u^2$,

tj. $u \leq \frac{1}{4}$.

Primjer 2. - nastavak

- y je realan i ima dva rješenja:
  Za $u = 1/8$, $y_1 = 0{,}988$,
  $y_2 = -0{,}238$.