# Fourierova transformacija

# Fourierova transformacija kontinuiranog signala

Definicija:

$$X(F) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi Ft} dt$$

$$x(t) = \int_{-\infty}^{\infty} X(F) e^{j2\pi Ft} dF$$

# Lema o reciprocitetu

- Ako je X(F) Fourierova transformacija signala x(t) tada je Fourierova transformacija signala X(t), signal x(-F)

$$x(t) \xleftrightarrow[\text{Transformacija}]{\text{Fourierova}} X(F)$$

$$X(t) \xleftrightarrow[\text{Transformacija}]{\text{Fourierova}} x(-F)$$

## FT – Diracove δ funkcije

$$X(F)=\int_{-\infty}^{\infty}\delta(t)e^{-j2\pi ft}dt=1$$

- Prema lemi o reciprocitetu vrijedi

$$\delta(F)=\int_{-\infty}^{\infty}e^{-j2\pi ft}dt$$

## FT – sin(t)

- Poznato je da vrijedi $\sin t=\frac{e^{jt}-e^{-jt}}{2j}$

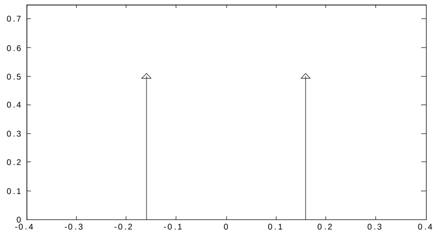
$$X(F)=\int_{-\infty}^{\infty}\sin(t)e^{-j2\pi Ft}dt=\int_{-\infty}^{\infty}\frac{e^{jt}-e^{-jt}}{2j}e^{-j2\pi Ft}dt=$$

$$=\frac{1}{2j}\int_{-\infty}^{\infty}e^{-j2\pi\left(F-\frac{1}{2\pi}\right)t}dt-\frac{1}{2j}\int_{-\infty}^{\infty}e^{-j2\pi\left(F+\frac{1}{2\pi}\right)t}dt=$$

$$=\frac{1}{2j}[\delta(F-\frac{1}{2\pi})-\delta(F+\frac{1}{2\pi})]$$

## FT – sin(t)

- Konačno amplitudni spektar izgleda $|X(F)|=\frac{1}{2}[\delta(F-\frac{1}{2\pi})+\delta(F+\frac{1}{2\pi})]$



## FT – 10 perioda funkcije sin(t)

$$X(F)=\int_0^{20\pi}\sin(t)e^{-j2\pi Ft}dt=\int_0^{20\pi}\sin(t)[(\cos(2\pi Ft)-j\sin(2\pi Ft)]dt=$$

$$=\int_0^{20\pi}[\sin(t)\cos(2\pi Ft)-j\sin(t)\sin(2\pi Ft)]dt=$$

$$=\int_0^{20\pi}\sin(t)\cos(2\pi Ft)dt-j\int_0^{20\pi}\sin(t)\sin(2\pi Ft)dt=$$

$$=\frac{1}{2}\int_0^{20\pi}[\sin((1+2\pi F)t)+\sin((1-2\pi F)t)]dt+\frac{1}{2}j\int_0^{20\pi}[\cos((1+2\pi F)t)-\cos((1-2\pi F)t)]dt=$$

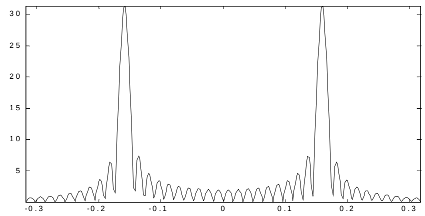
$$=\frac{1-\cos(40\pi^2F)}{1-4\pi^2F^2}+j\frac{\sin(40\pi^2F)}{1-4\pi^2F^2}$$

## FT – 10 perioda funkcije sin(t)

- Konačno amplitudni spektar izgleda

$$|X(F)|=\left|\frac{2\sin(20\pi^2F)}{1-4\pi^2F^2}\right|=\left|\frac{\sin(20\pi^2F)}{1-2\pi F}+\frac{\sin(20\pi^2F)}{1+2\pi F}\right|=$$

$$=\left|\frac{\sin[20\pi^2(F+\frac{1}{2\pi})]}{2\pi(F+\frac{1}{2\pi})}-\frac{\sin[20\pi^2(F-\frac{1}{2\pi})]}{2\pi(F-\frac{1}{2\pi})}\right|$$




## FT – 1 periode funkcije sin(t)

- Koristimo teorem o konvoluciji

$$X(F)=\int_{-\pi}^{\pi}e^{-j2\pi Ft}dt=2\pi\frac{\sin 2\pi^2F}{2\pi^2F}$$ Spektar pravokutnog impulsa

$$X(F)=\frac{1}{2j}[\delta(F-\frac{1}{2\pi})-\delta(F+\frac{1}{2\pi})]$$ Spektar beskonačnog sinusa

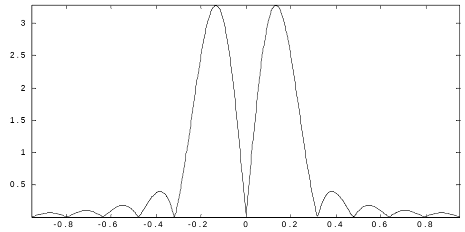
$$X(F)=\int_{-\infty}^{\infty}2\pi\frac{\sin 2\pi^2\tau}{2\pi^2\tau}\frac{1}{2j}[\delta(F-\tau+\frac{1}{2\pi})-\delta(F-\tau-\frac{1}{2\pi})]d\tau=$$

$$=\frac{\sin 2\pi^2(F+\frac{1}{2\pi})}{j2\pi(F+\frac{1}{2\pi})}-\frac{\sin 2\pi^2(F-\frac{1}{2\pi})}{j2\pi(F-\frac{1}{2\pi})}$$

# FT – 1 periode funkcije sin(t)

- Konačno amplitudni spektar izgleda

$$|X(F)| = \left| \frac{\sin[2\pi^2(F+\frac{1}{2\pi})]}{2\pi(F+\frac{1}{2\pi})} - \frac{\sin[2\pi^2(F-\frac{1}{2\pi})]}{2\pi(F-\frac{1}{2\pi})} \right|$$




# Svojstva Fourierove transformacije diskretnih signala

prije je izvedena transformacija za aperiodični diskretni signal (DTFT)

$$X(e^{j\omega}) \equiv F\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$x[n] \equiv F^{-1}\{X(e^{j\omega})\} = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

ovdje se razmatraju neka svojstva DTFT pogodna u nizu praktičnih primjena

slična razmatranja vrijede i za kontinuirane signale

# Svojstva Fourierove transformacije diskretnih signala

<u>svojstvo linearnosti</u>

ako je $X_1(e^{j\omega}) = \mathrm{F}\{x_1[n]\}$ i $X_2(e^{j\omega}) = \mathrm{F}\{x_2[n]\}$ tada je $X(e^{j\omega}) = \mathrm{F}\{\alpha x_1[n] + \beta x_2[n]\} = \alpha X_1(\omega) + \beta X_2(\omega)$

$$X(e^{j\omega}) \equiv F\{\alpha x_1[n] + \beta x_2[n]\} = \sum_{n=-\infty}^{\infty}\left(\alpha x_1[n] + \beta x_2[n]\right)e^{-j\omega n}$$

$$= \alpha\sum_{n=-\infty}^{\infty} x_1[n]e^{-j\omega n} + \beta\sum_{n=-\infty}^{\infty} x_2[n]e^{-j\omega n} =$$

$$= \alpha X_1(e^{j\omega}) + \beta X_2(e^{j\omega})$$

# Svojstva Fourierove transformacije diskretnih signala

pomak u vremenskoj domeni

ako je $X(e^{j\omega}) = \mathrm{F}\{x[n]\}$ tada je

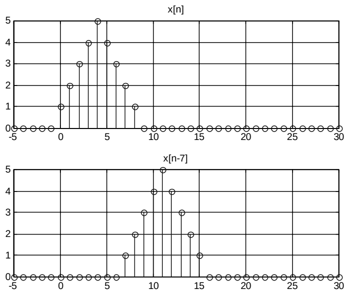
$\mathrm{F}\{x[n-k]\} = e^{-j\omega k} X(e^{j\omega})$

$$F\{x[n-k]\} = \sum_{n=-\infty}^{\infty} x[n-k]e^{-j\omega n} =$$

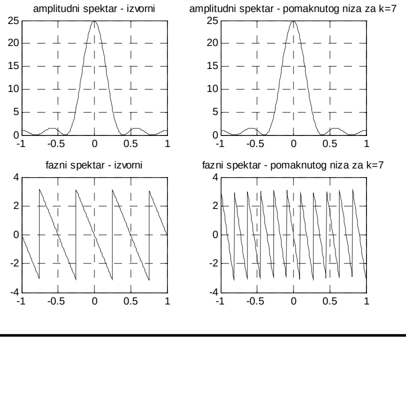
$$= \left| n-k=m \right| = \sum_{m=-\infty}^{\infty} x[m]e^{-j\omega(m+k)} =$$

$$= e^{-j\omega k} \sum_{m=-\infty}^{\infty} x[m]e^{-j\omega m} = e^{-j\omega k} X(e^{j\omega})$$

# *Svojstva Fourierove transformacije diskretnih signala*




# *Svojstva Fourierove transformacije diskretnih signala*



## *Svojstva Fourierove transformacije diskretnih signala*

<u>pomak u frekvencijskoj domeni</u>

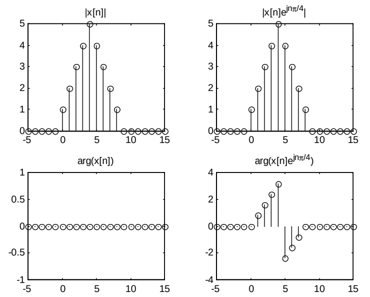
ako je $X(e^{j\omega}) = \mathrm{F}\{x[n]\}$ tada je

$$F\{e^{j\omega_0 n}x[n]\} = X(e^{j(\omega-\omega_0)})$$

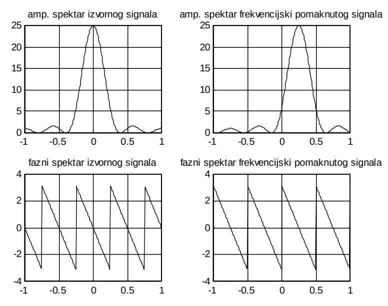
$$F\{e^{j\omega_0 n}x[n]\} = \sum_{n=-\infty}^{\infty} e^{j\omega_0 n}x[n]e^{-j\omega n} =$$

$$= \sum_{n=-\infty}^{\infty} x[n]e^{-j(\omega-\omega_0)n} = X(e^{j(\omega-\omega_0)})$$

## Svojstva Fourierove transformacije diskretnih signala




## Svojstva Fourierove transformacije diskretnih signala



## *Svojstva Fourierove transformacije diskretnih signala*

konvolucija

ako je $X_1(e^{j\omega}) = \mathrm{F}\{x_1[n]\}$ i $X_2(e^{j\omega}) = \mathrm{F}\{x_2[n]\}$ tada je

$$\mathrm{F}\left\{\sum_{m=-\infty}^{\infty} x_1[m]\cdot x_2[n-m]\right\} = X_1(e^{j\omega})\cdot X_2(e^{j\omega})$$

$$F\left\{\sum_{m=-\infty}^{\infty} x_1[m]x_2[n-m]\right\} = \sum_{n=-\infty}^{\infty}\left[\sum_{m=-\infty}^{\infty} x_1[m]x_2[n-m]\right]e^{-j\omega n} =$$

$$= \sum_{m=-\infty}^{\infty} x_1[m]\left[\sum_{n=-\infty}^{\infty} x_2[n-m]e^{-j\omega n}\right] = |n-m=l| =$$

## Svojstva Fourierove transformacije diskretnih signala

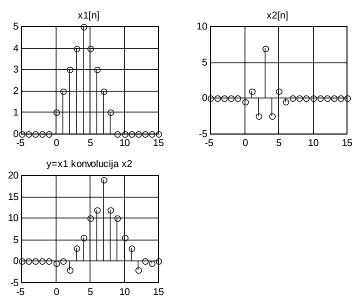
$$= \sum_{m=-\infty}^{\infty} x_1[m]\left[\sum_{l=-\infty}^{\infty} x_2[l]e^{-j\omega(m+l)}\right] =$$

$$= \sum_{m=-\infty}^{\infty} x_1[m]e^{-j\omega m}\left[\sum_{l=-\infty}^{\infty} x_2[l]e^{-j\omega l}\right] =$$

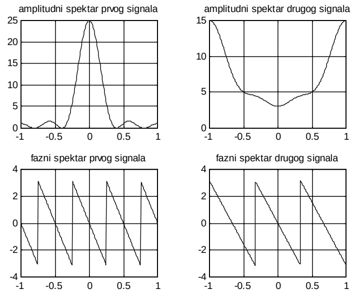
$$= X_2(e^{j\omega})\cdot \sum_{m=-\infty}^{\infty} x_1[m]e^{-j\omega m} =$$

$$= X_2(e^{j\omega})\cdot X_1(e^{j\omega})$$

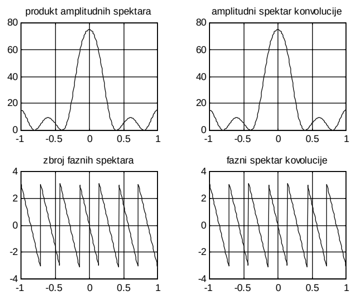
## Svojstva Fourierove transformacije diskretnih signala



## Svojstva Fourierove transformacije diskretnih signala




## Svojstva Fourierove transformacije diskretnih signala




## *Svojstva Fourierove transformacije diskretnih signala*

modulacija

ako je $X_1(e^{j\omega}) = \mathrm{F}\{x_1[n]\}$ i $X_2(e^{j\omega}) = \mathrm{F}\{x_2[n]\}$ tada je

$$\mathrm{F}\{x_1[n]\cdot x_2[n]\} = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(e^{j\psi})\cdot X_2(e^{j(\omega-\psi)})d\psi$$

$$\mathrm{y[n]} = x_1[n]\cdot x_2[n] \Rightarrow Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty}\left(x_1[n]x_2[n]\right) e^{-j\omega n}$$

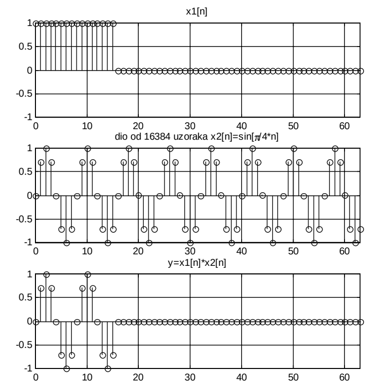
$$x_1[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(e^{j\omega})e^{j\omega n}d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(e^{j\psi})e^{j\psi n}d\psi$$

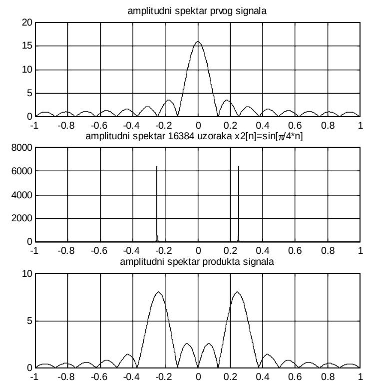
## *Svojstva Fourierove transformacije diskretnih signala*

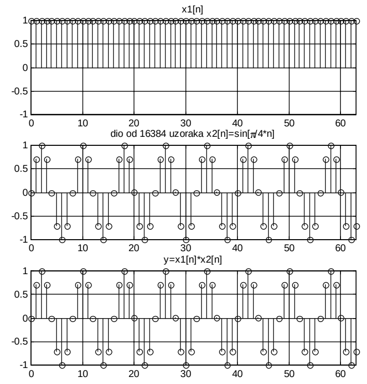
$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \left[ \frac{1}{2\pi} \int_{-\pi}^{\pi} X_1(e^{j\psi}) e^{j\psi n} d\psi \right] x_2[n] e^{-j\omega n}$$

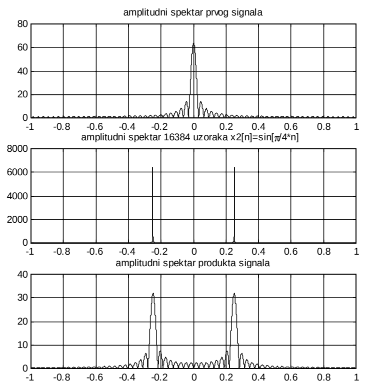
$$Y(e^{j\omega}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X_1(e^{j\psi}) \underbrace{\sum_{n=-\infty}^{\infty} x_2[n] e^{-j(\omega-\psi)n}}_{X_2(e^{j(\omega-\psi)})} d\psi$$

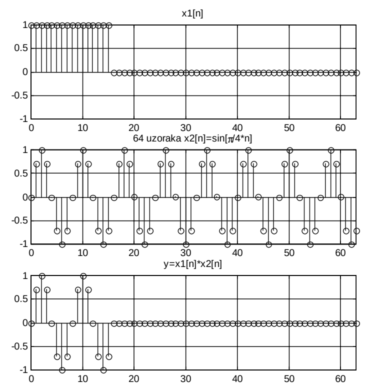
$$Y(e^{j\omega}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X_1(e^{j\psi}) X_2(e^{j(\omega-\psi)}) d\psi = \mathrm{F}\{x_1[n] \cdot x_2[n]\}$$

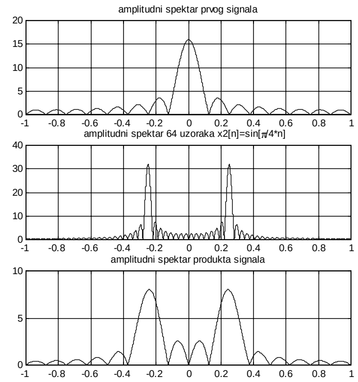


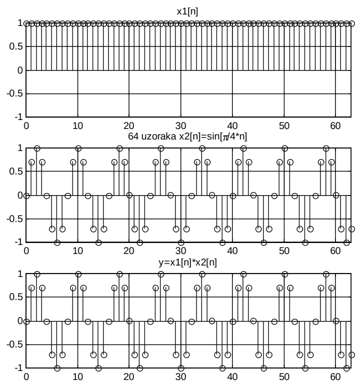



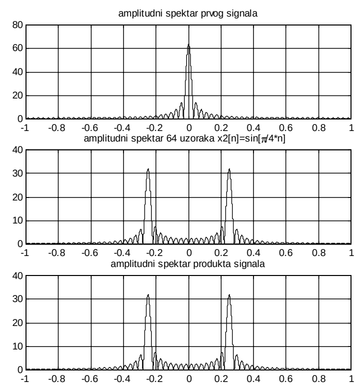

x1[n]
dio od 16384 uzoraka x2[n]=sin[π/4*n]
y=x1[n]*x2[n]

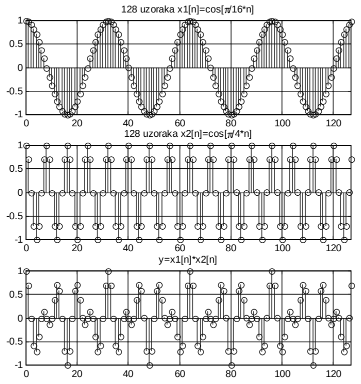

amplitudni spektar prvog signala
amplitudni spektar 16384 uzoraka x2[n]=sin[π/4*n]
amplitudni spektar produkta signala


x1[n]
64 uzoraka x2[n]=sin[π/4*n]
y=x1[n]*x2[n]


amplitudni spektar prvog signala
amplitudni spektar 64 uzoraka x2[n]=sin[π/4*n]
amplitudni spektar produkta signala


x1[n]
64 uzoraka x2[n]=sin[π/4*n]
y=x1[n]*x2[n]

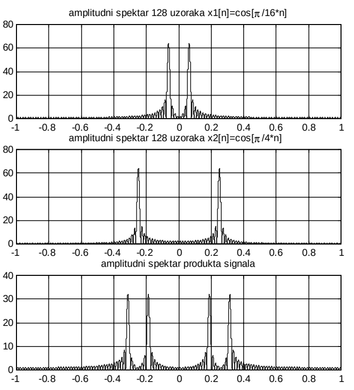

amplitudni spektar prvog signala
amplitudni spektar 64 uzoraka x2[n]=sin[π/4*n]
amplitudni spektar produkta signala




# primjer

$$x_1[n] = \cos[\tfrac{\pi}{4} n]$$
$$x_2[n] = \cos[\tfrac{\pi}{16} n]$$
$$x_1[n] \cdot x_2[n] = \cos[\tfrac{\pi}{4} n] \cdot \cos[\tfrac{\pi}{16} n] =$$

$$x_1[n] \cdot x_2[n] = \tfrac{1}{2}\left\{\cos[\tfrac{3\pi}{16} n] + \cos[\tfrac{5\pi}{16} n]\right\} =$$




1/16=
0.0625

1/4=
0.25

3/16=
0,1875

5/16=
0,3125

# Svojstva Fourierove transformacije diskretnih signala

inverzija vremenske osi

ako je $X(e^{j\omega}) = \mathrm{F}\{x[n]\}$ tada je $X(e^{-j\omega}) = \mathrm{F}\{x[-n]\}$

$$F\{x[-n]\} = \sum_{n=-\infty}^{\infty} x[-n] e^{-j\omega n} = \left| n = -l \right| =$$

$$= \sum_{l=-\infty}^{\infty} x[l] e^{j\omega l} = X(e^{-j\omega})$$

# Svojstva Fourierove transformacije diskretnih signala

za realni $x[n]$ dobivamo:

$$F\{x[-n]\} = X(e^{-j\omega}) = \left| X(e^{-j\omega}) \right| e^{j \arg\left\{X(e^{-j\omega})\right\}}$$

$$= \left| X(e^{j\omega}) \right| e^{-j \arg\left\{X(e^{j\omega})\right\}}$$